STUDY OF SCATTERING PARAMETERS OF WAVEGUIDE DEVICES USING COMSOL MULTYPHYSICS

A thesis submitted in partial fulfillment of the requirements for the degree of Bachelor of Science in Electrical, Electronics & Communication Engineering

Submitted by

Tanjila Ahmed          Student No. 200616027
Samiul Islam          Student No. 200616028
Shajid Islam          Student No. 200616031

Supervised by

Dr. Md. Shah Alam
Associate Professor
Department of Electrical and Electronics Engineering
Bangladesh University of Engineering & Technology
Dhaka-1000, Bangladesh
Declaration

It is hereby declared that this thesis or any part of it has not been submitted elsewhere for the award of any degree or diploma.

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(Tanjila Ahmed)           (Samiul Islam)             (Shajid Islam)

Supervisor

Dr. Md. Shah Alam
Associate Professor,
Department of Electrical and Electronic Engineering,
Bangladesh University of Engineering and Technology (BUET),
Dhaka 1000, Bangladesh.
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Abstract

S-parameters refer to the scattering matrix. The concept was first popularized around the time that Kaneyuke Kurokawa of Bell Labs wrote his 1965 IEEE article Power Waves and the Scattering Matrix. It helped that during the 1960s, Hewlett Packard introduced the first microwave network analyzers. The scattering matrix is a mathematical construct that quantifies how RF energy propagates through a multi-port network. The S-matrix is what allows us to accurately describe the properties of incredibly complicated networks as simple "black boxes. One difficulty associated with transmission parameters is that in a practical high-frequency circuit it is extremely difficult to obtain the perfect open-circuit and short-circuit conditions required to make good measurements. The S-parameter method circumvents the open/short difficulties by always connecting both ports to transmission lines with stable, well-defined impedances. To the extent that such a test setup better represents the actual working conditions of your circuit, the S-parameter method generates a more accurate model. These are the reasons why this thesis topic is chosen. In this thesis paper, an idea about what is S-parameters, it’s classification and usefulness is described elaborately. Then here the mathematical and network based analysis is done which helps to know about S-parameters more accurately. Here S-parameters are expressed with mathematical equations. Examples are being done by the help of the software named FEMLAB. FEMLAB is a powerful, interactive environment for modeling and solving scientific and engineering problems based on partial differential equations. It has helped to know the detailed status of waves in different types of waveguides. These detailed statuses are being described at the end of this thesis.
Chapter 1

Introduction

1.1 S-parameter

Scattering parameters or S-parameters are properties used in electrical engineering, electronics engineering, and communication systems engineering describing the electrical behavior of linear electrical networks when undergoing various steady state stimuli by small signals. S-parameters are a mathematical subset of a more general non-linear formulation called X-parameters.

An electrical network to be described by S-parameters may have any number of ports. Ports are the points at which electrical currents either enter or exit the network. Sometimes these are referred to as pairs of 'terminals'. So for example, a 2-port network is equivalent to a 4-terminal network, though this terminology is unusual with S-parameters, because most S-parameter measurements are made at frequencies where coaxial connectors are more appropriate. S-parameters describe the response of an N-port network to voltage signals at each port. The first number in the subscript refers to the responding port, while the second number refers to the incident port. Thus $S_{21}$ means the response at port 2 due to a signal at port 1. The most common "N-port" in microwaves is one-ports and two-ports. Three-port network S-parameters are easy to model with software such as Agilent ADS, but the three-port S-parameter measurements are extremely difficult to perform with accuracy. The S-parameter matrix describing an N-port network will be square of dimension 'N' and will therefore contain $N^2$ elements. At the test frequency each element or S-parameter is represented by a unit less complex number, thus representing magnitude and angle, or amplitude and phase. The complex number may either be expressed in rectangular form or, more commonly, in polar form. The S-parameter magnitude may be expressed in linear form or logarithmic form. When expressed in logarithmic form, magnitude has the "dimensionless unit" of decibels. The S-parameter angle is most frequently expressed in degrees but occasionally in radians.
Any $S$-parameter may be displayed graphically on a polar diagram by a dot for one frequency or a locus for a range of frequencies. If it applies to one port only (being of the form $S_{nn}$), it may be displayed on an impedance or admittance Smith Chart normalized to the system impedance. The Smith Chart allows simple conversion between the $S_{nn}$ parameter, equivalent to the voltage reflection coefficient and the associated (normalized) impedance (or admittance) seen at that port. The following information must be defined when specifying any $S$-parameter:

- The characteristic impedance (often 50 $\Omega$).
- The allocation of port numbers.
- Conditions which may affect the network, such as frequency, temperature, control voltage, and bias current, where applicable.

When we are talking about networks that can be described with $S$-parameters, we are usually talking about single-frequency networks. Receivers and mixers are not referred to as having $S$-parameters, although we can certainly measure the reflection coefficients at each port and refer to these parameters as $S$-parameters. The trouble comes when we wish to describe the frequency-conversion properties, this is not possible using $S$-parameters.

$S$-parameters are members of a family of similar parameters used in electronics engineering, other examples being: Y-parameters, Z-parameters, H-parameters, T-parameters or ABCD-parameters. They differ from these, in the sense that $S$-parameters do not use open or short circuit conditions to characterize a linear electrical network; instead matched loads are used. These terminations are much easier to use at high signal frequencies than open-circuit and short-circuit terminations. Moreover, the quantities are measured in terms of power. Many electrical properties of networks or components may be expressed using $S$-parameters, such as gain, return loss, voltage standing wave ratio (VSWR), reflection coefficient and amplifier stability.

The term 'scattering' is more common to optical engineering than RF engineering, referring to the effect observed when a plane electromagnetic wave is incident on an obstruction or passes across dissimilar dielectric media. In the context of $S$-parameters, scattering refers to the way in which the traveling currents and voltages in a transmission line are affected when they meet a discontinuity caused by the insertion of a network into
the transmission line. This is equivalent to the wave meeting impedance differing from the line's characteristic impedance. Although applicable at any frequency, S-parameters are mostly used for networks operating at radio frequency (RF) and microwave frequencies where signal power and energy considerations are more easily quantified than currents and voltages. S-parameters change with the measurement frequency so this must be included for any S-parameter measurements stated, in addition to the characteristic impedance or system impedance. S-parameters are readily represented in matrix form and obey the rules of matrix algebra.

Historically, an electrical network would have comprised a 'black box' containing various interconnected basic electrical circuit components or lumped elements such as resistors, capacitors, inductors and transistors. For the S-parameter definition, it is understood that a network may contain any components provided that the entire network behaves linearly with incident small signals. It may also include many typical communication system components or 'blocks' such as amplifiers, attenuators, filters, couplers and equalizers provided they are also operating under linear and defined conditions.

1.2 Classification

There are different types of S-parameters which are:

- **Small signal S-parameter**
  They are what we are talking about 99% of the time. By small signal, we mean that the signals have only linear effects on the network, small enough so that gain compression does not take place. For passive networks, small-signal is all we have to worry about, because they act linearly at any power level.

- **Large signal S-parameter**
  They are more complicated. In this case, the S-matrix will vary with input signal strength.

- **Mixed-mode S-parameter**
  They refer to a special case of analyzing balanced circuits.
• **Pulsed S-parameter**

They are measured on power devices so that an accurate representation is captured before the device heats up.

### 1.3 Objective of thesis

Network scattering parameters are powerful tools for the analysis and design of high frequency and microwave networks. Scattering Parameters, or s-parameters, are the reflection and transmission coefficients between the incident and reflection waves. They describe completely the behavior of a device under linear conditions at microwave frequency range. Each parameter is typically characterized by magnitude, decibel and phase. The s-parameters are voltage ratios of the waves. The advantage of s-parameters does not only lie in the complete description of the device performance at microwave frequencies but also the ability to convert to other parameters such as hybrid (H) or admittance (Y) parameters. Additionally, stability factor (K) and many gain parameters can be computed also.

In this thesis we have chosen a software named FEMLAB to find out S-parameter of microwave devices. FEMLAB is a powerful, interactive environment for modeling and solving scientific and engineering problems based on partial differential equations. It can easily extend conventional models that address one branch of physics to state-of-the art multiphysics models that simultaneously involve multiple branches of science and engineering. Multiphysics treat simulations that involve multiple physical models or multiple simultaneous physical phenomena. For example, combining chemical kinetics and fluid mechanics or combining finite elements with molecular dynamics. Multiphysics typically involve solving coupled systems of partial differential equations. Accessing this power, however, does not require an in-depth knowledge of mathematics or numerical analysis. Indeed, one can build many useful models simply by defining the relevant physical quantities rather than defining the equations directly. FEMLAB then internally compiles a set of PDEs representing the problem. FEMLAB also allows creating equation-based models. When solving the PDEs that describe a model, FEMLAB applies the finite element method (FEM). FEMLAB runs that method in conjunction with adaptive meshing and error control as well as with a variety of numerical solvers.
Working in an easy-to-use graphical interface or from a command line, users choose from several ways to describe their problems in 1D, 2D, and 3D. A particular strength of this software is its PDE modeling capability, enabling equations from various fields such as structural mechanics, electromagnetics, fluid flow, and chemistry to be linked and solved all in the same model and all at the same time. These and many other features make FEMLAB an unprecedented modeling environment for research, product development and education.

1.4 Organization of thesis

This thesis consists of four chapters. A brief description of each chapter is given below.

In chapter 1, here is the definition and short description of $S$-parameters. A classification of $S$-parameters regarding its signal condition is also mentioned here. By the end of this chapter the objective of this thesis and its organization pattern is being described.

In chapter 2, here is a network based analysis of $S$-parameters. Its mathematical representation in terms of power flow is also derived here. Need of calculation of $S$-parameters, its PDE formulation for TE and TM waves are described at the end.

In chapter 3, process of using COMSOL Multiphysics 3.2 is noted performing two examples. One is waveguide H-bend and the other one is waveguide adapter.

In chapter 4, the possible results of the two examples described in the previous chapter are given. Along with these results individual discussions are given also.

In chapter 5, here is the conclusion of this thesis. A brief idea about what is done so far in the thesis is given in this conclusion part.
Chapter 2

Mathematical Description of $S$-Parameters

2.1 Network based description

$S$-parameters are a set of parameters describing the scattering and reflection of traveling waves when a network is inserted into a transmission line. $S$-parameters are normally used to characterize high frequency networks, where simple models valid at lower frequencies cannot be applied. The scattering matrix is a mathematical construct that quantifies how RF energy propagates through a multi-port network. For an RF signal incident on one port, some fraction of the signal bounces back out of that port, some of it scatters and exits other ports and is perhaps even amplified, and some of it disappears as heat or even electromagnetic radiation.

$S$-parameters are normally measured as a function of frequency, so when looking at the formulae for $S$-parameters it is important to note that frequency is implied, and that the complex gain (i.e. gain and phase) is also assumed. For this reason, $S$-parameters are often called complex scattering parameters.

![Fig. 2.1 A two port network.](image)

A two-port device shown in Fig. 2.1 can be described by a number of parameter sets. We’re familiar with the $H$-, $Y$-, and $Z$-parameter sets. All of these network parameters
relate total voltages and total currents at each of the two ports. All these parameters are applicable in lower frequency. We cannot apply these parameters for the analysis of higher frequency because moving to higher and higher frequencies, some problems arise:

- Equipment is not readily available to measure total voltage and total current at the ports of the network.
- Short and open circuits are difficult to achieve over a broad band of frequencies.
- Active devices, such as transistors and tunnel diodes, very often will not be short or open circuit stable.

Some method of characterization is necessary to overcome these problems. The logical variables to use at these frequencies are traveling waves rather than total voltages and currents and thus the concept of $S$-parameters come. As we said earlier that in high frequency we consider waves (both traveling and reflected) instead of voltage and current. Where reflected waves are the function of incident waves. Let us look at $E_{r2}$ in Fig. 2.2 where we see that it is made up of that portion of $E_{i2}$ reflected from the output port of the network as well as that portion of $E_{i1}$ that is transmitted through the network. Each of the other waves are similarly made up of a combination of two waves.

![Fig. 2.2 High frequency two port network.](image)

It should be possible to relate these four traveling waves by some parameter set. While the derivation of this parameter set will be made for two-port networks, it is applicable for $n$-ports as well. Let’s start with the $H$-parameter set for a two port network. We can write
\[ V_1 = h_1 I_1 + h_2 V_2 \]  
\[ I_2 = h_2 I_1 + h_2 V_2 \]  
(2.1)  
(2.2)

For the Fig.2.2, we can write for the total current and total voltage

\[ V_1 = E_{i1} + E_{r1} \]  
\[ V_2 = E_{i2} + E_{r2} \]  
(2.3)  
(2.4)

\[ I_1 = \frac{E_{i1} - E_{r1}}{Z_0} \]  
\[ I_2 = \frac{E_{i2} - E_{r2}}{Z_0} \]  
(2.5)  
(2.6)

By substituting the expressions for total voltage and total current on a transmission line into this parameter set, we can rearrange these equations such that the incident traveling voltage waves are the independent variables; and the reflected traveling voltage waves are the dependent variables.

\[ E_{r1} = f_{11}(h)E_{i1} + f_{12}(h)E_{i2} \]  
\[ E_{r2} = f_{21}(h)E_{i1} + f_{22}(h)E_{i2} \]  
(2.7)  
(2.8)

The functions \( f_{11}, f_{21}, f_{12}, f_{22} \) represent a new set of network parameters relating traveling voltage waves rather than total voltages and total currents. It is appropriate that we call this new parameter set “\( S \)-parameters,” since they relate those waves scattered or reflected from the network to those waves incident upon the network. These scattering parameters will commonly be referred to as \( S \)-parameters. So now the equation becomes

\[ E_{r1} = S_{11}E_{i1} + S_{12}E_{i2} \]  
\[ E_{r2} = S_{21}E_{i1} + S_{22}E_{i2} \]  
(2.9)  
(2.10)

Now let us divide the above equation by \( \sqrt{Z_0} \), the characteristic impedance of the transmission line, the relationship becomes

\[ b_1 = S_{11}a_1 + S_{12}a_2 \]  
\[ b_2 = S_{21}a_1 + S_{22}a_2 \]  
(2.11)  
(2.12)

Here is the matrix algebraic representation of 2-port \( S \)-parameters
\[
\begin{pmatrix}
  b_1 \\
  b_2 
\end{pmatrix} = 
\begin{pmatrix}
  S_{11} & S_{12} \\
  S_{21} & S_{22} 
\end{pmatrix} \times 
\begin{pmatrix}
  a_1 \\
  a_2 
\end{pmatrix}
\]  \tag{2.13}

where the incident waves are designated by the letter \( a_n \), where \( n \) is the port number of the network. For each port, the incident (applied) and reflected wave are measured. The reflected wave is designed by \( b_n \), where \( n \) is the port number.

\[
a_1 = \frac{E_{i1}}{Z_0} = \frac{\text{wave incident on port } 1}{\sqrt{Z_0}}
\]  \tag{2.14}

\[
a_2 = \frac{E_{i2}}{Z_0} = \frac{\text{wave incident on port } 2}{\sqrt{Z_0}}
\]  \tag{2.15}

\[
b_1 = \frac{E_{r1}}{Z_0} = \frac{\text{wave reflected from port } 1}{\sqrt{Z_0}}
\]  \tag{2.16}

\[
b_2 = \frac{E_{r2}}{Z_0} = \frac{\text{wave reflected from port } 2}{\sqrt{Z_0}}
\]  \tag{2.17)

\[S_{11} = \left. \frac{b_1}{a_1} \right|_{a_2 = 0} = \text{Input reflection coefficient with the output port terminated by a matched load (} Z_L = Z_o \text{ sets } a_2 = 0 \).\]

\[S_{22} = \left. \frac{b_2}{a_2} \right|_{a_1 = 0} = \text{Output reflection coefficient with the input port terminated by a matched load (} Z_s = Z_o \text{ sets } a_1 = 0 \).\]

\[S_{21} = \left. \frac{b_2}{a_1} \right|_{a_2 = 0} = \text{Forward transmission gain with the input port terminated in a matched load (} a_2 = 0 \).\]

\[S_{12} = \left. \frac{b_1}{a_2} \right|_{a_1 = 0} = \text{Reverse transmission gain with the input port terminated in a matched load (} a_1 = 0 \).\]

Now, we need to calculate s-parameters by the means of electric field corresponding to the voltage. To convert an electric field pattern on a port to a scalar complex number corresponding to the voltage in transmission line theory we need to perform an eigenmode expansion of the electromagnetic fields on the ports. Let us assume that we have performed eigenmode analyses on the ports 1, 2, 3 and so on and that we know the
electric field patterns \(E_1, E_2, E_3, \ldots\) of the fundamental modes on these ports. The fields should have zero phases. Furthermore, let us assume them to be normalized with respect to the integral of the power flow across each port cross section respectively. Let us note that this normalization is frequency dependent unless we are dealing with TEM modes. The port excitation is assumed to be of unity power and applied using the fundamental eigenmode. The computed electric field \(E_c\) on the port consists of the excitation plus the reflected field. The S-parameters is given by

\[
S_{11} = \frac{\int_{\text{port1}} ((E_c - E_1)E_1^*)dA_1}{\int_{\text{port1}} (E_1E_1^*)dA_1} \quad (2.18)
\]

\[
S_{21} = \frac{\int_{\text{port2}} ((E_cE_2^*)dA_2}{\int_{\text{port2}} (E_2E_2^*)dA_2} \quad (2.19)
\]

\[
S_{31} = \frac{\int_{\text{port3}} ((E_cE_3^*)dA_3}{\int_{\text{port3}} (E_3E_3^*)dA_3} \quad (2.20)
\]

**S-parameters in terms of power flow:**

It is possible to calculate the S-parameters from the power flow through the ports. Such a definition is only the absolute value of the S-parameters which does not have any phase information. The advantage of using the power flow to calculate the S-parameters is that the modes need not to be known. When the modes are degenerated, as for example is the case for circular ports, it is generally impossible to know how the wave is oriented. It is impossible to find the correct superposition of the degenerated modes to match the calculated wave. First we see the relation between the variables \(a_1, a_2, b_1, b_2\) and various power waves.
\[ |a_1|^2 = \text{power incident on the input of the network} \]
\[ = \text{power available from a source of impedance } Z_0 \]
\[ |a_2|^2 = \text{power incident on the output of the network} \]
\[ = \text{power reflected from the load} \]
\[ |b_1|^2 = \text{power reflected from the input port of the network} \]
\[ = \text{power available from a } Z_0 \text{ source minus the power delivered to the input of the network} \]
\[ |b_2|^2 = \text{power reflected or emanating from the output of the network} \]
\[ = \text{power incident on the load} \]
\[ = \text{power that would be delivered to a } Z_0 \text{ load} \]

Hence $S$-parameters are simply related to power gain and mismatch loss, quantities which often are of more interest than the corresponding voltage function.

\[ |S_{11}|^2 = \frac{\text{power reflected from the network input}}{\text{power incident on the network input}} \]
\[ |S_{22}|^2 = \frac{\text{power reflected from the network output}}{\text{power incident on the network output}} \]
\[ |S_{21}|^2 = \frac{\text{power delivered to a } Z_0 \text{ load}}{\text{power available from } Z_0 \text{ source}} \]
\[ = \text{transducer power gain with } Z_0 \text{ load and source} \]
\[ |S_{12}|^2 = \text{reverse transducer power gain with } Z_0 \text{ load and source} \]

$S$-parameter magnitudes are presented in one of two ways, linear magnitude or decibels (dB). Because $S$-parameters are a voltage ratio, the formula for decibels in this case is

\[ S_y (dB) = 20 \times \log[S_y (\text{magnitude})] \]  \hfill (2.21)

### 2.2 Requirement of calculation of scattering parameters

Scattering parameters mainly describes an electrical network. More specifically, many electrical properties of networks may be expressed using scattering parameters; such as, gain, return loss, voltage standing wave ratio (VSWR), reflection coefficient and amplifier stability. It is a way to describe the operation of a linear, time invariant two-port circuit. Scattering parameters are very popular as a way to characterize connectors and
cables for high-speed applications above Gbps. Moreover, some problems occur when not using scattering parameters:

- A broadband open or short circuit is difficult to achieve at high frequencies.
- Active devices very often will oscillate when terminated in a reactive load.
- At high frequencies, total voltage and total current cannot be measured directly.

Scattering parameters, however, are measured with resistive terminations and are determined by measuring incident and reflected waves.

2.3 PDE formulation for plane waves

In order to calculate the $S$-parameters of a microwave network, we need to find the partial differential equation solving which we can calculate our requirements. For PDE formulation in case of plane waves, we consider a situation where we have no variation in the $z$ direction, and the electromagnetic field propagates in the modeling $x$-$y$ plane. The application mode can handle:

- Transverse electric (TE) waves.
- Transverse magnetic (TM) waves.
- Hybrid-mode waves. A hybrid-mode wave is simply a linear combination of a TE and a TM wave. Working with hybrid-mode waves allows for specifying elliptical or circular waves as incoming waves.

We will only consider TE and TM waves here in the following sub sections.

2.3.1 PDE formulation for the TE waves

As the field propagates in the modeling $x$-$y$ plane, a TE wave has only one electric field component in the $z$ direction, and the magnetic field lies in the modeling plane. Thus the transient and time-harmonic fields can be written

$$E(x,y,t) = E_z(x,y,t) = E_z(x,y)e^{j\omega t}$$  \hspace{1cm} (2.22)

$$H(x,y,t) = H_x(x,y,t)e^z + H_y(x,y,t)e^y = (H_x(x,y)e^x + H_y(x,y)e^y)e^{j\omega t}$$  \hspace{1cm} (2.23)
To be able to write the fields in this form, it is also required that \( \varepsilon_r, \sigma, \) and \( \mu_r \) are non-diagonal only in the \( x-y \) plane. \( \mu_r \) denotes a 2-by-2 tensor, and \( \varepsilon_{xx} \) and \( \sigma_{zz} \) are the relative permittivity and conductivity in the \( z \) direction. Given the above constraints, the equation is

\[
\nabla \times (\mu_r^{-1} \nabla \times \mathbf{E}) - k_0^2 \varepsilon_{rr} \mathbf{E} = 0
\]

(2.24)

where

\[
\varepsilon_{rr} = \varepsilon_r - j \frac{\sigma}{\omega \varepsilon_0}
\]

(2.25)

can be simplified to a scalar equation for

\[
E_z - \nabla \cdot (\tilde{\mu}_r \nabla E_z) - \varepsilon_{rrrr} k_0^2 E_z = 0
\]

(2.26)

where

\[
\tilde{\mu}_r = \frac{\mu_r \tau}{\det(\mu_r)}
\]

(2.27)

Using the relation \( \varepsilon_r = n^2 \), where \( n \) is the refractive index, the equation can alternatively be written

\[- \nabla \cdot \nabla E_z - n_{zz}^2 k_0^2 E_z = 0
\]

(2.28)

when the equation is written using the refractive index, the assumption is that \( \mu_r = 1 \) and \( \sigma = 0 \).

The wave number in vacuum \( k_0 \) is defined by

\[
k_0 = \omega \sqrt{\mu_0 \varepsilon_0} = \frac{\varepsilon}{c_0}
\]

(2.29)

where \( c_0 \) is the speed of light in vacuum. The equation in the time-domain

\[
\mu_0 \sigma \frac{\delta A}{\delta t} + \mu_0 \varepsilon_0 \frac{\delta}{\delta t} (\varepsilon_r \frac{\delta A}{\delta t} - \mathbf{D}_r) + \nabla \times (\mu_r^{-1} (\nabla \times \mathbf{A} - \mathbf{B}_r)) = 0
\]

(2.30)

can be simplified to a scalar equation for \( A_z \),

\[
\mu_0 \sigma \frac{\delta A_z}{\delta t} + \mu_0 \varepsilon_0 \frac{\delta}{\delta t} (\varepsilon_r \frac{\delta A_z}{\delta t} - D_z) + \nabla \times (\mu_r^{-1} (\nabla \times A_z - \mathbf{B}_z)) = 0
\]

(2.31)
Here, the constitutive relations used are \( B = \mu_0 \mu_r H + B_r \) and \( D = \mu_0 \mu_r E + D_r \). Other constitutive relations can also be handled for transient problems. Using the relation \( \varepsilon_r = n^2 \), where \( n \) is the refractive index, the equation can alternatively be written

\[
\frac{\mu_0 \varepsilon_0}{\delta t} \left( n^2 \frac{\delta A_z}{\delta t} + \nabla (\nabla A_z - B_r) \right) = 0
\]

(2.32)

when using the refractive index, the assumption is that \( \mu_r = 1 \) and \( \sigma = 0 \) and only the constitutive relations for linear materials can be used.

### 2.3.2 PDE formulation for TM waves

TM waves have a magnetic field with only a \( z \) component and an electric field in the \( x-y \) plane. Thus the fields can be written as

\[
H(x, y, t) = H_z(x, y, t) = H_z(x, y) e_z e^{j\omega t}
\]

(2.33)

\[
E(x, y, t) = E_x(x, y, t) e_x + E_y(x, y,t) e_y = (E_x(x, y) e_x + E_y(x, y) e_y) e^{j\omega t}
\]

(2.34)

To write the fields in this form, it is also required that \( \mu_r \) and \( \varepsilon_r \) are nondiagonal only in the \( x-y \) plane. \( \varepsilon_r \) and \( \sigma \) denote 2-by-2 tensors, and \( \mu_{zz} \) is the relative permeability in the \( z \) direction. Given the above constraints the time-harmonic equation

\[
\nabla \times (\varepsilon_{zz}^{-1} \nabla \times H) - k_0^2 \mu_r H = 0
\]

(2.34)

can be simplified to a scalar equation for

\[
H_z - \nabla (\tilde{\varepsilon}_r \nabla H_z) - \mu_{zz} k_0^2 H_z = 0
\]

(2.35)

where

\[
\tilde{\varepsilon}_r = \frac{\varepsilon_{zz}^{-1}}{\det(\varepsilon_{zz})}
\]

(2.36)

Using the relation \( \varepsilon_r = n^2 \), where \( n \) is the refractive index, the equation can alternatively be written

\[
-\nabla (n^2 \nabla H_z) - k_0^2 H_z = 0
\]

(2.37)

In the time-domain the equation for TM wave equation is simplified to

\[
\frac{\mu_0 \sigma}{\delta t} \frac{\delta A}{\delta t} + \frac{\mu_0}{\delta t} (\varepsilon_r \frac{\delta A}{\delta t} - D_r) + \nabla \times (\mu_r^{-1} (\nabla \times A - B_r)) = 0
\]

(2.38)
Here, the constitutive relations $\mathbf{B} = \mu_0 \mu_r \mathbf{H} + \mathbf{B}_r$ and $\mathbf{D} = \mu_0 \mu_r \mathbf{E} + \mathbf{D}_r$ are used. Other constitutive relations can also be handled by the application mode. Using the relation $\varepsilon_r = n^2$, where $n$ is the refractive index, the equation can alternatively be written

$$\mu_0 \varepsilon_0 \frac{\delta}{\delta t} (n^2 \frac{\delta \mathbf{A}}{\delta t}) + \nabla \times (\nabla \times \mathbf{A}) = 0$$

(2.39)

In this case only the constitutive relations for linear materials can be used.
Chapter 3

Use of COMSOL Multiphysics

A very useful method to calculate the scattering parameter is Finite Element Method. Again, COMSOL Multiphysics 3.2 is an efficient software to calculate the scattering parameters using Finite Element Method. We have carried out three different tutorials on COMSOL Multiphysics 3.2 for different types of waveguides and saw the results. Different steps of the tutorials are shown below.

3.1 Waveguide H-bend

Model Library path: Electromagnetics_Module/
RF_and_Microwave_Engineering/hbend_3d

3D Modeling Using the Graphical User Interface:

Model navigator:
1. In the Model Navigator, 3D was selected in the Space dimension list.
2. In the list of application modes, Electromagnetics Module>Electromagnetic Waves>Harmonic propagation was selected.
3. OK was clicked.

Options and settings:

![Fig. 3.1 Constants dialogue box.](image)
Table 3.1 Table of constants

<table>
<thead>
<tr>
<th>Expression</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fc</td>
<td>4.3e9</td>
</tr>
<tr>
<td>fq1</td>
<td>1.2×fc</td>
</tr>
</tbody>
</table>

Geometry modeling:

1. From the Draw menu, the Work Plane Settings dialog box was opened. OK was clicked to obtain the default workplane in the x-y plane.
2. From the Options menu, Axes/Grid Settings was chosen.
3. In the Axes/Grid Settings dialog box, the following axis and grid settings were specified:

Table 3.2 Axes/Grid settings

<table>
<thead>
<tr>
<th>Axis</th>
<th>Grid description</th>
<th>Grid settings</th>
</tr>
</thead>
<tbody>
<tr>
<td>x min</td>
<td>-0.175</td>
<td>x spacing 0.05</td>
</tr>
<tr>
<td>x max</td>
<td>0.175</td>
<td>Extra x 0.075-0.0174245</td>
</tr>
<tr>
<td>y min</td>
<td>-0.04</td>
<td>y spacing 0.05</td>
</tr>
<tr>
<td>y max</td>
<td>0.2</td>
<td>Extra y -0.0174245</td>
</tr>
</tbody>
</table>

The waveguide width 2*0.0174245 corresponds to a cut-off frequency of 4.3 GHz.
4. Three lines were drawn. After selecting the line tool, (0,-0.0174),
(-0.1,-0.0174), (-0.1, +0.0174), and (0, +0.0174) was clicked.
5. The 2nd Degree Bezier Curve button was clicked. After that (0.0576, 0.0174) and (0.0576, 0.075) were clicked.
6. The Line button was clicked. So was (0.0576, 0.175), (0.0924, 0.175), and (0.0924, 0.075).
7. The 2nd Degree Bezier Curve button was clicked. Then the point (0.0924,-0.0174) was clicked.
8. The right mouse button was clicked to close the boundary curve and create a solid object.
9. Extrude was selected from the Draw menu. The object was extruded using a distance of 0.0174.
10. The Zoom Extents toolbar button was clicked.
Physics Modeling:
Scalar Variables:
1. From the Physics menu, Scalar Variables was chosen.
2. In the Application Scalar Variables dialog box, the frequency \( \nu_{emw} \) was set to \( f_{q1} \).

Boundary Conditions:
1. From the Physics menu, Boundary Settings was chosen.
   1. Boundaries 2–8 and 10 were selected.
   2. In the Boundary condition list, perfect electric conductor was set as the boundary condition. These boundaries represent the inside of the walls of the waveguide which is plated with a metal, such as silver, and considered to be a perfect conductor.
   3. On boundaries number 1 and 9, the Port boundary condition was specified. On the Port tab the values according to the table 3.3 were set.

Sub domain Settings:
Default values for \( \varepsilon_r \), \( \mu_r \) and \( \sigma \) can be used because the waveguide is filled with air.

Mesh Generation:
1. In the Mesh Parameters dialog box 0.006 was typed in the Maximum element size field.

Fig. 3.4 Mesh parameters dialogue box.
2. Remesh was clicked to generate the mesh.

![Image](image.png)

Fig. 3.5 Three dimensional view after doing remesh.

**Computing the Solution:**

The Solve button was clicked to solve the problem.

**Post Processing and Visualization:**

The default plot is a slice plot of the z component of the electric field. To better see the propagating wave the position of the slices were changed.

1. On the Slice tab in the Plot Parameters dialog box the Slice positioning x levels was set to 0, the y levels to 0, and the z level to 1.
2. A convenient way to visualize the good transmission is to plot the total energy density. Reflections give rise to a wave pattern in the energy distribution. Total energy density, time average was selected as the slice data.

3. To compare the power flow of the incident wave to the power flow of the outgoing wave, boundary integration was performed. The entrance port was excited using the port boundary condition which automatically normalizes the excitation to unity power (1 W). To obtain the power outflow from the exit port, power outflow, time average was integrated over boundary 9. The result was 0.985795 W.

Table 3.3 Port boundary specifications

<table>
<thead>
<tr>
<th>Boundary</th>
<th>1</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Port number</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Wave excitation at this port</td>
<td>selected</td>
<td>cleared</td>
</tr>
<tr>
<td>Mode specification</td>
<td>rectangular</td>
<td>rectangular</td>
</tr>
<tr>
<td>Mode type</td>
<td>Transverse electric(TE)</td>
<td>Transverse electric(TE)</td>
</tr>
<tr>
<td>Mode number</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>
3.2 Waveguide adapter

Model Library path:
Electromagnetics_Module/RF_and_Microwave_Engineering/waveguide_adapter

Modeling Using the Graphical User Interface:
The waveguide adapter model is set up in two steps. First, the boundary mode analysis is performed on the elliptical port of the adapter; and then the wave propagation problem is solved using the mode shape as a boundary condition.

The file waveguide_adapter_geometry.mph was opened in the Model Library. The geometry in the file is a waveguide transition with one elliptical end and one rectangular end. The following steps were used to calculate the S-parameters for this geometry when the wave enters from the rectangular end and exits at the elliptical end.

1. The Model Navigator was opened from the Multiphysics menu.
2. In the Model Navigator the Electromagnetics Module> Boundary Mode Analysis> TE Waves application mode was selected and added it to the model.

3. Then the Electromagnetics Module> Electromagnetic Waves> Harmonic propagation application mode was selected and added it to the model.

4. OK was clicked to close the Model Navigator.

First, the Boundary Mode Analysis application mode was used to compute the eigenmode of the elliptical end. This eigenmode is then part of the boundary condition for the electromagnetic wave propagation simulation.

**Physics settings:**
First, the boundary mode analysis problem was set up. Boundary Mode Analysis was selected from the Multiphysics menu.

**Scalar Variables:**
In the Scalar Variables dialog box, the frequency \( \nu_{\text{emweb}} \) was set to \( 7 \times 10^9 \). This is a frequency above the cut-off frequency, which ensures that you find a propagating wave mode when doing the mode analysis.

**Boundary Settings:**
In the Boundary Settings dialog box, the Active in this domain check box was cleared for all boundaries except boundary 6. This boundary is the elliptical end of the waveguide, where the mode analysis will be done.

**Edge Settings:**
In the Edge Settings dialog box the Perfect electric conductor boundary condition was selected at edges 5, 6, 10, 14, 29, 39, 52 and 55. These are the exterior edges of the ellipse.

**Mesh generation:**
1. In the Mesh Parameters dialog box, 0.003 was entered in the Maximum element size edit field.
2. The mesh was initialized.

**Computing the solution:**
1. The Solver Parameters dialog box was opened.
2. On the General tab, the Eigenvalue solver was selected and entered 1 in the Search for effective mode indices around edit field.
3. OK was clicked to close the dialog box.
4. The Solver Manager dialog box was opened and selected only the Boundary Mode Analysis application mode on the Solve For tab.
5. The Solve button was clicked.

**Postprocessing and visualization:**
The eigenmodes were visualized as a boundary plot.
1. In the Plot Parameters dialog box, the Slice check box was cleared and selected the Boundary check box.
2. The Effective mode index was selected 0.535.
3. On the Boundary tab, the Tangential electric field, x component was selected. There are two field variables one for each of the two application modes. The tEx_emweb variable was selected.
4. OK was clicked to plot the field and the Go to XY view button was clicked to better see the field.
5. The same type of plot was made of the y component of the electric field, tEy_emweb.
6. To see the normal component of the magnetic field, Normal magnetic field (wev) was selected in the Predefined quantities list.
7. On the General tab, 90 was entered in the Solution at angle (phase) edit field. This compensates for the phase shift of 90 degrees between the electric and the magnetic field.
8. OK was clicked to see the plot.
Now the 3D problem was continued to set up to do the S-parameter analysis. Then Go to Default 3D View button was clicked in order to see the full geometry.

![Graph showing the y component of the electric field for the first eigenmode of the elliptical port.](image)

Fig. 3.8: The y component of the electric field for the first eigenmode of the elliptical port.

**Boundary Settings:**

1. Electromagnetic Waves was selected in the Multiphysics menu.
2. The Boundary Settings dialog box was opened.
3. At the rectangular end, boundary 13, the Port boundary condition was selected.

On the Port tab the values were set according to the table below:

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Port no.</td>
<td>1</td>
</tr>
<tr>
<td>Wave excitation at this port</td>
<td>Selected</td>
</tr>
<tr>
<td>Mode specification</td>
<td>Rectangular</td>
</tr>
<tr>
<td>Mode type</td>
<td>Transverse Electric (TE)</td>
</tr>
<tr>
<td>Mode No.</td>
<td>10</td>
</tr>
</tbody>
</table>
4. At the elliptical end, boundary 6, the Port boundary condition was selected. On the Port tab, the values were set according to the table 3.5.

Table 3.5 Port boundary specifications for port 2

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Port no.</td>
<td>2</td>
</tr>
<tr>
<td>Wave excitation at this port</td>
<td>Cleared</td>
</tr>
<tr>
<td>Mode specification</td>
<td>Numeric</td>
</tr>
<tr>
<td>Mode type</td>
<td>Automatic</td>
</tr>
<tr>
<td>Use numeric data from</td>
<td>Boundary mode analysis, TE waves (wev)</td>
</tr>
</tbody>
</table>

5. The default Perfect electric conductor boundary condition was selected at all other boundaries.

**Computing the solution:**

1. The Solver Manager dialog box was opened. On the Initial Value tab, Store Solution was clicked. In the dialog box that appeared, the solution was clicked with the effective mode index 0.535 and OK was clicked.
2. The Initial Value was set to stored solution.
3. On the Solve For tab only the Electromagnetic Waves application mode was selected.
4. OK was clicked to close the Solver Manager.
5. In the Solver Parameters dialog box, the Parametric linear solver was selected.
6. Name of parameter was set to freq and List of parameter values to linspace(6.6e9,1e10,50).
7. In the Linear system solver list, Direct (SPOOLES) was selected.
8. The Symmetric was selected in the Matrix symmetry list in order to save memory.
9. On the Advanced tab, the Use Hermitian transpose check box was cleared.
10. OK was clicked to close the Solver Parameters dialog box.
11. The Application Scalar Variables dialog box was opened from the Physics menu and the Synchronize equivalent variables check box was cleared. This allows us to set different values for the two frequency variables nu_emwev and nu_emw.
nu_emweb is the frequency variable for the Boundary Mode Analysis application mode. It is important that it retains its value of 7e9 in order for the data from the mode analysis solution to be evaluated correctly. The frequency variable nu_emw was set to freq. This is the frequency variable of the 3D Electromagnetic Waves application mode.

12. The eigenvalue variable lambda from the mode analysis solution is not available in postprocessing after solving the second problem. Therefore we needed to define a constant lambda with the value -6169. This lambda value corresponds to the effective mode index 0.535. Lambda was defined in the Constants dialog box. To find the value of lambda, we could also use the Data Display dialog box after doing the mode analysis but before solving the second problem.

13. The Solve button was clicked.

Postprocessing and visualization:

1. To visualize the x component of the electric field in the waveguide, the Plot Parameters dialog box was opened.
2. On the General tab, the Solution at angle (phase) text field reads 0 was made sure.
3. The Boundary check box was cleared, and selected the Slice check box.
4. The Slice tab was clicked and Electric field, x component (weh) was selected in the Predefined quantities list.
5. Under Slice Positioning, the number of x levels, y levels, and z levels were set to 1, 1, and 0 respectively.
6. OK was clicked to see the plot.

The port boundary conditions generate two S-parameter variables, S11_emw and S21_emw as well as two variables S11dB_emw and S21dB_emw, which are the S-parameters on a dB scale. The names of the variables are derived from the port numbers entered in the Boundary Settings dialog box. A domain plot was made in a point to plot the S-parameters as functions of frequency.

1. On the Point tab in Domain Plot Parameters dialog box point 1 was selected and S-parameter dB (S11) (emw) was selected. Apply was clicked to plot S11 on a dB scale.
2. When plotting S21 first the two first data points were deselected on the General tab to better see the variation. The variable S-parameter dB (S21) (emw) was selected and OK was selected to make the plot.
Chapter 4

Results and Discussion

There were many parameters which can be found using the COMSOL MULTIPHYSICS for the three dimensional waveguide H- bend, waveguide adapter and three-port ferrite circulator. Some of them are shown below:

4.1 Waveguide H-bend

This example illustrates how to create a model that computes the electromagnetic fields and transmission characteristics of a 90-degree bend for a given radius. This type of waveguide bends changes the direction of the H field components and leaves the direction of the E field unchanged. The waveguide is therefore called an H-bend. Here, the side of coordinate -.1 indicates the input port while the side of coordinate .15 indicates the output port. In the input, a signal of 1 watt is applied in all the cases of this experiment. This model examines a TE wave, one that has no electric field component in the direction of propagation. More specifically, for this model we select the frequency
and waveguide dimension so that $TE_{10}$ is the single propagating mode. In that mode the electric field has only one nonzero component—a sinusoidal with two nodes, one at each of the walls of the waveguide. One important design aspect is how to shape a waveguide to go around a corner without incurring unnecessary losses in signal power.

The output figure of the analysis and explanation of the figures are given below:

In Fig. 4.2, we can see that the electric field component passes between top and bottom of the waveguide. It is maximum at centre and zero at conducting walls so the energy must flow through center of the waveguide.

The magnetic field forms closed path surrounding the electric displacement currents so the magnetic field is maximum at the boundary as shown in the Fig. 4.3.

The Fig. 4.4 shows the time average of total energy density of the waveguide. Total energy should be maximum at the centre while zero at the walls as we used here perfectly electric conductor as boundary condition to maintain the loss minimum. The figure also shows that with this type of design and cutoff frequency the objective was achieved.

In the Fig. 4.5, we can see the electric field in z direction. The applied wave is TE wave which has electric field component in the direction of propagation which is z direction. It is seen in the figure that the variation happens as a cosine function.

In the Fig. 4.6 we can see the x component of the magnetic field. As the direction of propagation is in z direction and magnetic field is perpendicular with electric field so we can see the sinusoidal variation of magnetic field with perpendicular of electric field. This is the figure of x component so the magnetic field in y direction is not clearly visible.

In the Fig. 4.7 we can see the y component of the magnetic field. As the direction of propagation is in z direction and magnetic field is perpendicular with electric field so we
can see the sinusoidal variation of magnetic field with perpendicular of electric field. This is the figure of y component so the magnetic field in x direction is not clearly visible.

In the Fig. 4.8, we can see for x component the power exists in x direction as we bend the waveguide in y direction there are no power other than x direction and the same result also seen from the y direction in Fig. 4.9.

In the Fig. 4.10, electric field displacement in z direction is shown. As the electric field moves by bouncing the walls of the waveguide so we see in the figure that there once occur positive pick and then negative peak as cosine wave which is expected.

The Fig 4.11 shows the surface current density in z component. We know that this is the direction of propagation and the input is TE10 wave. So there is no magnetic field component in z direction and as a result we didn’t see any current in the upper and lower surface (though lower surface is not visible in the figure). There are current component in two side of the structure as it follows the magnetic field.

The Fig 4.12 shows the surface current density normalized which includes both x and y component of the surface current. So we can see the surface current circulates in the total surface and as there is no surface current in z component so the normalized value of the surface current in the center is low to some extent.

In Fig. 4.13, the boundary plot of z component of tangential electric field of the waveguide is shown. The waveguide walls are typically plated with a very good conductor, such as silver. In this example the walls are considered to be made of a perfect conductor, implying that $\mathbf{n} \times \mathbf{E} = 0$ on the boundaries. This boundary condition is referred to as a perfect electric conductor (PEC) boundary condition.
Fig. 4.2 Time average of electrical energy density taking a slice of the waveguide.

Fig. 4.3 Time average of magnetic energy density taking a slice of the waveguide.
Fig. 4.4 Time average of total energy density taking a slice of the waveguide.

Fig. 4.5 Z component of the electric field taking a slice of the waveguide.
Fig. 4.6 X component of magnetic field taking a slice of the waveguide.

Fig. 4.7 Y component of the magnetic field taking a slice of the waveguide.
Fig. 4.8 Time average of the x component of power flow taking a slice of the waveguide.

Fig. 4.9 Time average of the y component of power flow taking a slice of the waveguide.
Fig. 4.10 Subdomain plot of z component of electric displacement.

Fig. 4.11 Boundary plot of z component of surface current density.
Fig. 4.12 Boundary plot of normal component of surface current density.

Fig. 4.13 Boundary plot of z component of tangential electric field.
4.2 Waveguide adapter

This is a model of an adapter for microwave propagation in the transition between a rectangular and an elliptical waveguide. Such waveguide adapters are designed to keep energy losses due to reflections at a minimum for the operating frequencies. To investigate the characteristics of the adapter, the simulation includes a wave traveling from a rectangular waveguide through the adapter and into an elliptical waveguide. The S-parameters are calculated as functions of the frequency. The involved frequencies are all in the single-mode range of the waveguide, that is, the frequency range where only one mode is propagating in the waveguide.

The output figure of the analysis and explanation of the figures are given below:

In the Fig. 4.14, we can see the boundary of the elliptical part which is passive but the eigenmode is still used in the boundary condition. We excite the input port with $TE_{10}$ wave. So in the direction of propagation there is only electric field. As perfect electric conductor in the boundary is used, so we found the maximum electric power in the center which is shown in the cross-section of the elliptical surface.

In the Fig. 4.15, we can see the magnetic energy density. We know magnetic field always flows perpendicularly with the electric field. So as electric energy density is in the center of the adapter, the magnetic energy density is maximum at the sides which are expected.

In the Fig. 4.16, the total energy density is shown. As the electric field is traveling in the direction of propagation and there is no electric field in the boundaries so it is expected that the total energy will flow through the center. This is what is verified by the figure. As the energy density flows through the center, so the power also flows in the figure which is shown in the Fig. 4.17.
In the Fig. 4.18, as the magnetic field flows perpendicularly with the electric field and here electric field flows with the direction of propagation, so the magnetic field is perpendicular with that. As the electric field varies sinusoidally, so the magnetic field also varies sinusoidally as in the figure shown the magnetic field occurs in two sides with maximum and minimum value.

In the Fig 4.19, we can see the normal component of magnetic flux density of the waveguide which is maximum in the input port (rectangular side) and at a lower value at the other side due to some loss. It is to be noted that we consider here only dielectric loss and no conductor loss ($\sigma = 0$).

In the Fig 4.20, we can see the normal component of electric field of the waveguide which is maximum in the input port (rectangular side) and at a lower value at the other side due to changes of mode in elliptical surface. It is to be noted that we consider here only dielectric loss and no conductor loss ($\sigma = 0$).

In the Fig. 4.21, we see the normal component of time average of power flow taking a slice of the waveguide. The magnitude of power is the maximum in the rectangular part of the waveguide. The magnitude of power flow is less in the elliptical side due to change of mode cause here it works in a hybrid mode.

In the Fig. 4.22, we see the sub domain plot of time average of electric energy density. This is the maximum in the rectangular side in the center as TE10 mode is there and showing lower value in elliptical part as there propagating hybrid mode of the wave. The corresponding magnetic field density was shown in the Fig. 4.23.

In Fig. 4.24, we see the sub domain plot of z component of magnetic flux density. As the electric field bounces in the x direction so there is a magnetic field travelling sinusoidally in the z direction as shown in the figure which is graphically same with z component of magnetic field shown in the Fig. 4.26.
Fig. 4.14 Boundary plot of time average of electric energy density.

Fig. 4.15 Boundary plot of time average of magnetic energy density.
Fig. 4.16 Boundary plot of time average of total energy density.

Fig. 4.17 Boundary plot of time average of normal power flow.
Fig. 4.18 Boundary plot of normal magnetic field.

Fig. 4.19 Normal component of magnetic flux density taking a slice of the waveguide.
Fig. 4.20 Normal component of electric field taking a slice of the waveguide.

Fig. 4.21 Normal component of time average of power flow taking a slice of the waveguide.
Fig. 4.22 Subdomain plot of time average of electric energy density.

Fig. 4.23 Subdomain plot of time average of magnetic energy density.
Fig. 4.24 Subdomain plot of z component of magnetic flux density.

Fig. 4.25 Subdomain plot of normal component of electric field.
Fig. 4.26 Subdomain plot of z component of magnetic field.

In the Fig. 4.25, we see the sub domain plot of electric field, norm. We know electric field propagates by bouncing in x direction so the result in the figure is expected. The side walls also show that the component is zero there.

Result of the s-parameter analysis:

We have found the graph for two s-parameters with respect to frequency ($S_{11}$ and $S_{21}$) which are shown below. For $S_{11}$, we can see that the S-parameter is the lowest at certain frequency and there are some other lower values of $S_{11}$ at certain frequencies. Similarly, for $S_{21}$, there are certain frequencies where the S-parameter is high. We can use these frequencies as windows for transmission as they involve lower loss.
Fig 4.27 $S_{11}$ versus Frequency.
Fig 4.28 $S_{21}$ versus Frequency.
Chapter 5

Conclusion

A scattering matrix (S-parameter matrix) is one way to describe the operation of a linear, time-invariant two-port circuit. In this thesis paper it was aimed to develop a basic and elaborate idea about $S$-parameters. The $S$-parameter matrix is rapidly becoming very popular as a way to characterize connectors and cables for high-speed applications above 1 Gb/s. Many electrical properties of networks or components may be expressed using $S$-parameters, such as gain, return loss, voltage standing wave ratio (VSWR), reflection coefficient and amplifier stability. As for high frequency analysis and microwave networks there is no alternative of $S$-parameters so it is must to study about it. At the beginning of this paper $S$-parameters with there short history and a significant classification is being mentioned. Then it was tried to find out the reason why we have to study about $S$-parameters and what is the basic differences with other network parameters. The notation which is used to represent a $S$-parameter of N-port network and it’s significant is mentioned in this very introductory part of this thesis. Our main objective of this thesis is described very particularly. FEMLAB is used to have a practical knowledge about what is actually happening in these waveguides and how the waves are being driven in them. . FEMLAB is a powerful, interactive environment for modeling and solving scientific and engineering problems based on partial differential equations. It can easily extend conventional models that address one branch of physics to state-of-the art multiphysics models that simultaneously involve multiple branches of science and engineering. For better understanding two examples are shown here. One is of waveguide H-bend and the other one is of waveguide adapter. The step by step process of doing the examples is mentioned here. Finally the results of these examples and probable description of these results are given in brief.
S-parameters are just one matrix that can fully describe a network. Other matrices include ABCD parameters, Y-parameters and Z-parameters. ABCD parameters are actually used "behind the scenes" in many calculations, because they are easily cascadable. By cascadable, we mean that if you want to simulate an attenuator followed by an amplifier, the S-parameter math will drive you insane, while the ABCD math involves nothing more than multiplication.
References


[8] www.sigcon.com/Pubs/news/6_03.htm
